TEMPORAL CONSTRAINTS BETWEEN CYCLIC GEOGRAPHIC EVENTS

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Abstract: This paper presents a data model for cyclic geographic events useful for next-generation geographic information system design. The paper introduces a structure that captures all possible temporal relations between occurrences of cyclic events, as well as the frequency in which each temporal relation occurs. Based on such a structure, this paper proposes a new set of temporal constraints and a methodology that selects from different temporal configurations of the cyclic events, the configuration that best represents the imposed temporal constraints.

Key words: Temporal reasoning; temporal constraints; cyclic constraints; cyclic events.

1. INTRODUCTION

The world is a dynamic place and for over a decade the geographic information science research community has had a strong interest in being able to capture dynamic or time-varying phenomena (Langran 1992). There has been a focus on developing spatio-temporal data models (Peuquet and Duan 1995; Abraham and Roddick 1999), including models that treat moving objects (Pfoser and Theodoridis 2003), and visualization of dynamic objects (Campos *et al.* 2003). Recently, there has been an effort to develop event-based approaches for geographic information science where events become the principal component of modeling (Worboys and Hornsby 2004).

Events are abstractions of interconnected phenomena and activities of the real world. These abstractions, however, do not represent standalone actions or behaviors. The happenings of events depend on the fulfillment of some conditions (i.e., prior or concurrent events) and imply some consequences in the modeled world. This interdependence between events combined with the complex nature of geographic phenomena imposes additional requirements for the conceptualization of a temporal data model. This model requires more elaborate temporal structures that capture the complexity of geographic events as well as a formal representation of the relationships between such events.

This paper considers a special category of events that include, for example, the movement of trains between cities, a person's daily routine of driving to work and then home again, or the movement of tides. These kinds of happenings occur repeatedly in a cyclic manner. Computational models need to be able to handle such *cyclic events* and the relationships among them. This paper focuses on the temporal characteristics of cyclic events and treats the relationships between events that can be expressed in terms of temporal constraints.

The remainder of this paper is structured as follows: Section 2 reviews the mechanism of temporal constraints between two intervals. Section 3 discusses the limitation of using temporal constraints between two intervals when dealing with cyclic events. Section 4 describes a structure to represent the set of temporal relations that hold between two cyclic intervals. Section 5 introduces instances of temporal constraints used to relate cyclic intervals and proposes a rationale to select the temporal arrangement of the cyclic intervals that best represent the intended constraints. Section 5 presents conclusions and indicates future work.

2. TEMPORAL CONSTRAINTS BETWEEN INTERVALS

A critical issue in the conceptualization of a temporal data model is the characterization of the fundamental temporal entity and the set of relations that holds between such entities. Many GIS applications adopt *temporal intervals* as the temporal representations of events and Allen's set of thirteen temporal relations as the *basic* relationships that hold between two intervals (Allen 1983). Choosing intervals as the temporal construct that corresponds to the time when an event is happening yields a simple but powerful abstraction of the temporal characteristics of events. These characteristics are represented by an interval's structural elements, that is, $Ev_i = (S_i, E_i)$, where Ev_i is the temporal representation of the event *i*, and S_i and E_i are the interval's start and end points, respectively.

Values assigned to the intervals' endpoints are called *unary temporal constraints* (Meiri 1996). These values may vary from a single absolute value (e.g., $S_i = 3$, $E_i = 5$) to complex expressions representing multiple values or ranges of values (e.g. $S_i = 5 \vee S_i = 7$, $E_i \geq 1 \wedge E_i \leq 3$). Complex expressions allow unary temporal constraints to capture, for example, uncertainties and incomplete knowledge about the temporal characteristics of events. These expressions, however, do not capture any temporal relationships that exist among events, valuable information that models the relative order between events (Frank 1998).

A common approach for relating two events is to impose a temporal relationship between two intervals. These relationships are called *binary temporal constraints* or *temporal constraints* for short. Thus, temporal constraints represent known relationships between events that occur in a geospatial domain.

The usual way of representing dependencies between temporal representations of two events is through triples of the form (Ev_i, Ev_j, τ). In these triples, the objects Ev_i and Ev_j are temporal representations of two events and the object τ is an instance of a temporal constraint that must hold between these intervals.

Instances of temporal constraints are any kind of information that relates two intervals. These instances can be divided into two groups: *qualitative* and *quantitative* temporal constraints. Qualitative constraints are represented by a disjunction of basic temporal relations. Thus, instances of such constraints may vary from values that impose a basic temporal relation to complex expressions involving disjunctions of many temporal relations. Instances of temporal constraints of the kinds `before` and `meets`, for example, correspond to imposing basic temporal relations between the intervals. Other instances of temporal constraints can be defined in order to represent more abstract relationships. A temporal constraint `startTogether`, for example, defines an instance of a temporal constraint in which only the start points of the intervals are constrained. This temporal constraint is represented by a disjunction of the temporal relations *starts* and *startedBy*. Quantitative constraints limit permissible values of the distance between two points of the related intervals. These constraints are often needed to capture relationships between geographic phenomena that cannot be expressed in terms of disjunction of temporal relations. The temporal constraint `centered`, for example, relates the midpoint of two event intervals. There is no disjunction of basic temporal relations able to capture such a configuration. Table 1 shows some triples of temporal constraints and the corresponding relationship that must hold between the endpoints of the related intervals.

Table 1. Triples of temporal constraints and the relationships between the intervals' endpoints

Temporal Constraints	Endpoint Relations
(Ev_i,Ev_j, `before`)	$S_i < S_j \wedge E_i < E_j$
(Ev_i,Ev_j, `meets`)	$S_i < S_j \wedge E_i = S_j$
(Ev_i,Ev_j, `startTogether`)	$S_i = S_j$
(Ev_i,Ev_j, `centered`)	$(S_i + E_i)/2 = (S_j + E_j)/2$

The temporal characteristics of events including their temporal constraints can be depicted in a graphical representation. Consider, for example, four events associated with an aircraft operation (i.e., *refueling*, *tire pressure checking*, *taxiing*, and *take-off*). Each event has a known duration and can be modeled as a temporal interval (Figure 1). Labeled arrows between intervals illustrate the temporal constraints. In this scenario, if the airplane needs to depart before the scheduled time, the other events must be rescheduled in order to guarantee that the plane does not depart without fuel or without the tire pressure being checked.

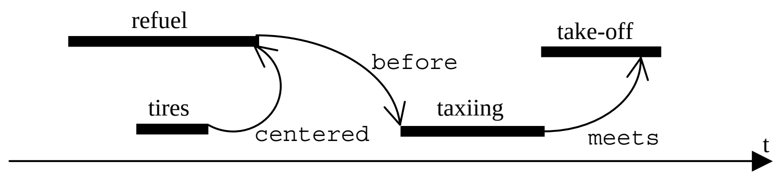

Figure 1. Representation of temporal constraints between events relating to an aircraft's operation.

The mechanism of representing temporal constraint between pairs of intervals to capture relationships between events falls short, however, when the events have a *cyclic pattern of repetition*. These kinds of event patterns require a more complex temporal model and a new set of temporal constraints that take into account the particularities of cycles. The next section discusses the limitations using instances of temporal constraints for two intervals to relate the temporal characteristics of cyclic events.

3. RELATING CYCLIC EVENTS WITH TEMPORAL CONSTRAINTS BETWEEN TWO INTERVALS

The temporal representation of cyclic events is a sequence of intervals that do not overlap. In such a representation, each interval corresponds to an occurrence of the cyclic event, called the cycle's *activity*. The gap between two occurrences of a cyclic event, if it exists, is referred to as the cycle's *inactivity*. The union of the cycle's activity and inactivity intervals defines the cycle's *period*. Consider, for example, a small portion of a timeline where occurrences of a cyclic event are depicted (Figure 2). The gray rectangles in the figure represent the cycle's occurrences or periods of activity and the white rectangles represent the temporal gap between two occurrences of the cycle. In this paper, we consider cycles with an infinite number of repetitions or with *frame-times* much larger than the cycles' period. *Frame-time* is an interval that encompasses all cycle occurrences (Terenziani 2003). The rationale discussed in this paper, however, can be easily extended to treat cycles that have only a finite number of repetitions.

The temporal characteristics of a cyclic event are represented by the structural elements of the collection of the cycle's occurrences and the gap between them, that is, $CycEv_i = (S_i, E_i, D)$, where $S_i = \{S_i^1, ..., S_i^n, ...\}$ and $E_i = \{E_i^1, ..., E_i^n, ...\}$ represent the collection of periods of activity's start and end points, and D is the duration of the period of *inactivity* (i.e., $S_i^{n+1} = E_i^n + D$).

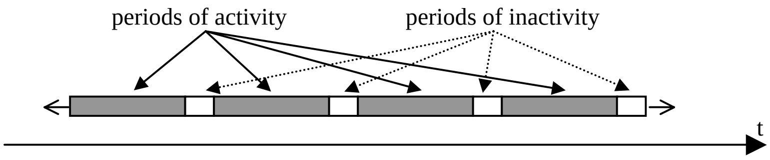

Figure 2. A segment of a timeline showing the temporal representation of a cyclic event.

Individual occurrences of a cyclic interval can be retrieved from this model. The k-th occurrence of the cyclic event *i*, for example, is defined as $CycEv_i^k = (S_i^k, E_i^k)$. In this way, it is possible to relate occurrences of cyclic intervals using instances of temporal constraints between two intervals (i.e., the mechanism for constraining pairs of intervals can be adapted to relate two cyclic intervals). The main modification is to allow elements in the triple of temporal constraints to be a certain occurrence of a cyclic interval.

Consider, for example, an application controlling the scheduling of two buses in an urban environment. Each bus travels a certain route many times and between each run, the buses rest for a certain amount of time. The temporal characteristics of the buses movement can be modeled as a sequence of intervals that do not overlap (i.e., cyclic intervals). Each interval represents the time needed for a bus to complete the route and the gap between the intervals corresponds to the break of the driver (Figure 3). Consider also that the two buses serve different routes in the financial district and the city has a policy that states that "whenever possible there is at least one bus running in the financial district". Taking into account the periods of activity and inactivity, it is possible to schedule these buses in a way that enforces the city's policy.

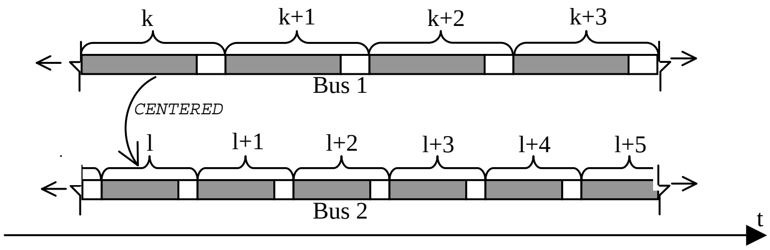

Figure 3. An arbitrary segment of a timeline showing the temporal representation of two cyclic events (Bus 1 and Bus 2) and a temporal constraint `centered`, which relates two occurrences of the cyclic intervals.

The temporal configuration shown in Figure 3 guarantees that there is always one bus running in the financial district. Thus, this configuration can be registered in the model through a temporal relationship between a run of each bus. One possible solution is to relate the k-th occurrence of *Bus 1* with the l-th occurrence of *Bus 2* with a temporal constraint that relates the midpoints of these occurrences, that is, $(CycEv^{k}_{Bus1}, CycEv^{l}_{Bus2}, \texttt{centered})$.

Imposing a temporal relationship between occurrences of each bus, however, is not robust enough to capture the knowledge from the application domain (i.e., the city policy concerning the schedule of the buses in the financial district). This relationship reflects only a solution found by a person that sets up the schedule of the buses based on the duration of each run. These kinds of temporal constraints limit the capability of the application to react to a change in the environment and to maintain the requirements of the application domain (i.e., known or desired relationships between cyclic events). For example, a new construction site will close off some streets in the route of *Bus 2*. This means that each run of this bus and the time between each run will be shorter (the duration of the rest of the driver is proportional to the duration of each run). The new temporal configuration of *Bus 2's* behavior and the fact that a temporal constraint relates only the midpoints of an occurrence of each cyclic interval no longer reflects the city's policy. Changing the duration of the periods of activity and inactivity of *Bus 2* and preserving the temporal relationship between two runs of the buses generates a temporal configuration where at some times, both buses are not running (Figure 4).

When two cycles are involved, a temporal relationship between two occurrences of cyclic intervals may not always be the most representative. Other occurrences will have different temporal relationships, with the possibility that the imposed relation is the less relevant (i.e., the relation occurs less frequently than other relations). Thus, a new set of instances of temporal constraints is needed to capture relationships that take into account temporal relations between *all occurrences* of a cyclic interval. A temporal constraint such as "*maximize the occurrence of periods of non-concurrent activities*", for example, gives the above application the information needed to find a new temporal configuration with at least one bus always running in the financial district. There is no guarantee, of course, that it will be possible to find such a temporal configuration, but the application can compute a configuration in which simultaneous periods of inactivity are distributed evenly during the day, minimizing the occurrence of long periods where no bus is running in the designed area.

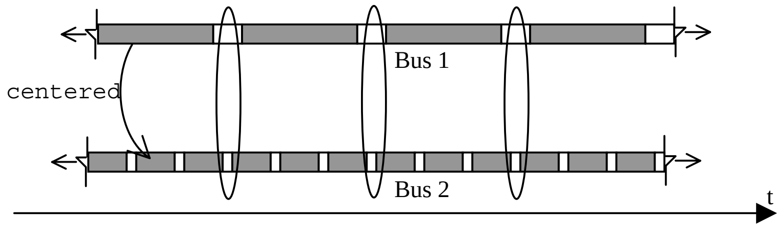

Figure 4. Graphical representations of the new temporal configuration of Buses 1 and 2. The oval highlights the period in which neither bus is running.

In order to treat temporal constraints between cyclic events, an exact account of the temporal relations between all occurrences of the cyclic intervals is needed. The next sections discuss the representation of temporal relations between cyclic intervals.

4. A REPRESENTATIVE SET OF TEMPORAL RELATIONS BETWEEN CYCLIC INTERVALS

Prior to treating temporal constraints between cyclic intervals, it is helpful to review some structural and ontological aspects of the underlying conceptualization of a model of time supporting cyclic events. Critical issues include:

- What are the possible temporal relations that hold between occurrences of two cyclic events?
- What is the temporal relation that occurs most often?

Previous studies have addressed the issue of temporal relations between collections of intervals. These studies can be divided into three main groups. The first group (Ladkin 1986; Leban *et al.* 1986; Balbiani *et al.* 1988; Morris and Khatib 1997) considers relations among generalized sequences of recurring events, that is, these studies do not consider any constraints in the formation of the elements of the sequence. The second group (Frank 1998; Hornsby *et al.* 1999; Osmani 1999) takes into account the cyclic pattern of the sequence of intervals but limit their scope to the particular case in which the cycles have the same period. The third group considers cycles with different periods (Cuckierman and Delgrande 2000; Terenziani 2003), but limit the representation of temporal relations to a disjunction of Allen's set of temporal relations (Allen 1983). This paper is more related with the second and third groups of study but our reasoning process depends on a more detailed representation of temporal relations that considers cycles with different periods as well as including the frequency of occurrence for each temporal relation that holds between occurrences of a cycle.

A temporal relation between occurrences of cyclic intervals is called *correlation* (Morris *et al.* 1996). Depending on the temporal characteristics of the cycle, the number of correlations can be infinite. After a certain amount of time, however, the pattern of correlations also repeats in a cyclic fashion. Thus, a finite subset of correlations can be chosen to represent all possible temporal relations that hold between occurrences of the cyclic intervals. We call this collection of temporal relations the *characteristic set of correlations*. The characteristic set of correlations is a multiset of correlations that capture all possible temporal relations between occurrences of cyclic intervals. If a certain correlation occurs more than once in the collection, the multiplicity of the element in the set must reflect the frequency of the temporal relation when all occurrences are considered. A characteristic set of correlations with the elements *contains*, *contains*, and *overlaps*, for example, reflects the fact that occurrences of the cyclic intervals are either under a temporal relation *contains* or *overlaps,* and that the relation *contains* occurs twice more frequently than the relation *overlaps*.

In order to obtain the characteristic set of correlations, it is necessary to compute all correlations over a specified amount of time. The minimum time required to capture all possible correlations as well as the frequency of each correlation is determined by the duration of the periods of the cycles involved in the process. We refer to this amount of time as the extended period of two cycles or the *period of equivalence* (E). The duration of the *period of equivalence* equals the least common multiplier between the duration of the cycles. Thus, the duration of *Bus 1's* period (D_1) and the duration of *Bus 2's* period (D_2) are used in the computation of the duration of the period equivalence (D_E) as follows: D_E=LCM(D_1, D_2). LCM is a function that returns the least common multiplier of two integers. Consider, for example, the initial temporal configuration of the buses behavior, that is, before some streets are closed (Figure 3). Based on the duration of the period of each bus behavior, the extent of the period of equivalence is computed. For this case, the period of equivalence is enough to cover two periods of *Bus 1* and three periods of *Bus 2* (Figure 5).

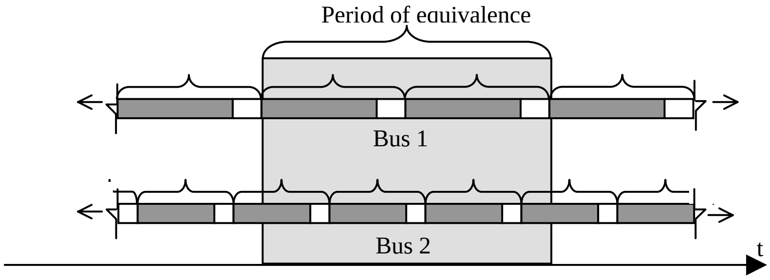

Figure 5. Occurrences of the Bus 1 and Bus 2 during the period of equivalence.

The duration of the period of equivalence defines the extent of a time window that captures all possible correlations between two cycles as well as the frequency in which each correlation occurs (i.e., the set of correlations inside the specified time window forms the characteristic set of correlations). The characteristic set of correlations is independent of the position of the time window along the timeline. Thus, cycles' occurrences that lie inside the time window can be seen as instances of classes that represent a certain number of consecutive occurrences of each cyclic interval. This abstraction allows the depiction of instances of these classes in a *cyclic representation* (Figure 6). In such a representation, the circumference of the circle equals the duration of the period of equivalence. A cyclic representation enhances the visualization of all correlations between cyclic intervals and facilitates the identification of the relations that form the characteristic set of correlations. Elements of the characteristic set of correlations are one out of twelve possible temporal relations between intervals in a cyclic representation, that is, the relations *before* and *after* are collapsed to a single relation called *disconnected* (Frank 1998; Hornsby *et al.* 1999). The characteristic set of correlations between *Bus 1* and *Bus 2*, for example, is composed of the temporal relations *overlaps*, *overlappedBy*, *metBy*, *contains*, and *meets* (Figure 6). All correlations refer to the temporal relation of an occurrence of the outer cyclic interval with respect to an occurrence of the inner cyclic interval read in the clockwise direction.

The temporal model for cyclic intervals is analogous to the Allen's (1983) temporal model for two linear intervals. Both models compare two temporal objects and describe a temporal relation that holds between these objects. In Allen's model of time, temporal objects are intervals related by one of thirteen basic temporal relations. In the temporal model for cyclic events, temporal objects are cyclic intervals (i.e., a collection of intervals) related by a set of temporal relations (i.e., a characteristic set of correlations). Different from Allen's temporal model, where the set of possible temporal relations is limited to thirteen, the set of possible temporal relations between two cyclic intervals can only be determined based on the durations of the cycles' periods and activity intervals. Thus, temporal reasoning involving cyclic intervals requires the identification of all different temporal relations between these entities. Different configurations of characteristic sets of correlations represent different temporal relations between two cyclic intervals. By different configurations, we mean two sets with the same number but different kinds of elements or two sets with different numbers of elements. With this approach, the order of the elements in the characteristic set of correlations is not relevant.

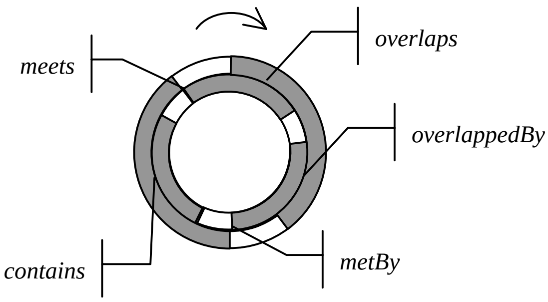

Figure 6. A cyclic representation of Bus 1 and Bus 2 and the corresponding characteristic set of correlations.

The process of determining all possible characteristic sets of correlations is accomplished by incrementing and decrementing the start point of one of the cycle's occurrences by one unit of time. In the linear representation (Figure 5), this maps to shifting one cyclic interval's occurrences by unitary increments in the positive or negative direction. In the cyclic representation (Figure 6), the same process is accomplished by rotating one cyclic interval's occurrences by angles equivalent to a unit of time in the clockwise or counter-clockwise direction. The number of increments or decrements needed to compute all different characteristic sets of correlations depends on the temporal characteristics of the cycles and on the granularity of the temporal space. For each increment or decrement, a new characteristic set of correlations is generated. Figure 7 depicts four possible temporal configurations between *Bus 1* and *Bus 2* and their respective characteristic sets of correlations. Others possible configurations of the characteristic set of correlations are possible but will give sets of correlations with the same elements in a different order and possibly with different qualitative characteristics (e.g., the amount of overlapping for occurrences of the relations *overlaps* and *overlappedBy*). These configurations are not treated in this paper.

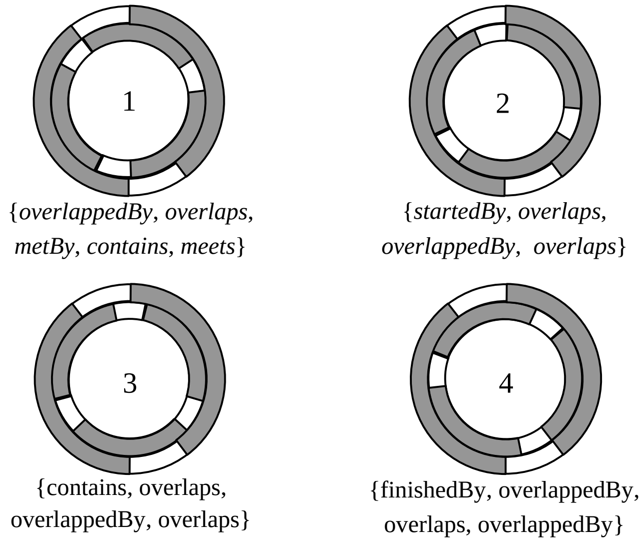

Figure 7. Possible characteristic sets of correlations between Bus 1 and Bus 2.

The set of all possible characteristic sets of correlations forms a model of temporal relations for two given cyclic events (i.e., there is no other set of correlations that may hold between these events). Once all possible temporal relations between cyclic events are known, it is possible to think about instances of temporal constraints that enforce a certain configuration among the set of possible characteristic set of correlations. Instances of such constraints as well as the rationale used to select a specific configuration are discussed in the next section.

5. TEMPORAL CONSTRAINTS WITH INSTANCES OF CYCLIC CONSTRAINTS

Characteristic sets of correlations provide valuable information that can be used to impose temporal constraints between two cyclic events. These sets inform not only all possible correlations between the cyclic intervals but also the frequency of occurrence of each correlation. Thus, instances of temporal constraints can exploit this kind of information and enforce a temporal configuration in which a certain correlation occurs more or less frequently.

We propose a new set of instances of temporal constraints for cyclic events based on the semantics of the characteristic sets of correlations. We call this set of values, *cyclic constraints*. Instances of such constraints are values of the form `maximizeRelation` and `mininimizeRelation`, where `Relation` assumes one of the following values, *disconnected*, *meets*, *overlaps*, *startTogether*, *containment*, *equal*, *finishTogether*, *overlappedBy*, or *metBy*. *Containment* is a temporal relation represented by the disjunction of the basic temporal relations *contains* and *containedBy*.

Based on instances of cyclic constraints, we propose a rationale to select, among all possible characteristic set of correlations, the configuration that best captures an intended constraint. The first criterion used in our rationale is to select a temporal configuration of the cyclic intervals in which the characteristic set of correlations has the largest (or smallest) number of a chosen relation. The first option is used for temporal constraints of the kind `maximizeRelation` while the second option is used for temporal constraints of the kind `mininimizeRelation`.

Consider, for example, all possible temporal configurations between occurrences of *Bus 1* and *Bus 2* depicted in Figure 7 and the city's policy of having always a bus running in the financial district. The requirement of the application domain can be represented, for example, as a cyclic temporal constraint of the form `maximizeDisconnected`. The idea here is to try to fill the gaps of periods of inactivity for one cyclic interval with periods of activity of the other cyclic interval. Given the set of instances of cyclic constraints, we consider that such a temporal constraint is the one that best reflects the requirement of a city concerned with bus schedules. The criterion of maximizing the temporal relation *disconnected*, however, is satisfied by all characteristic sets of correlations. Since the relation *disconnected* does not hold between occurrences of these cyclic intervals, all configurations have the same frequency (i.e., no occurrence of such a relation).

When more than one configuration satisfies an intended relation, we propose an additional criterion to break the tie. This second criterion is based upon the frequency of closest temporal relations (in a topological sense) that occur in the set of correlations. By closest temporal relations we mean the notion of conceptual neighborhood introduced by Freksa (Freksa 1992). Conceptual neighborhoods of temporal relations describe how similar two intervals are based on atomic deformations of one of the intervals. Figure 9 depicts the structure of the conceptual neighborhood for the set of relations between occurrences of cycles. Nodes in the graph represent temporal relations and the links connect conceptual neighborhood relations. In such a graph, the temporal relations *before* and *after* are collapsed into a single relation *disconnected*. Thus, the link between the two nodes representing the relation *disconnected* highlights the fact that the nodes represent the same relation and not a conceptual neighborhood. The graph in figure 8 is based on the structure of the b-neighborhood graph proposed by Freksa (Freksa 1992), which is obtained by the redefinition of an interval's start and end points in a way that preserves the length of the interval. We use such a graph because in our reasoning it is the same mechanism used to obtain different configurations of the characteristic set of correlation between two cycles.

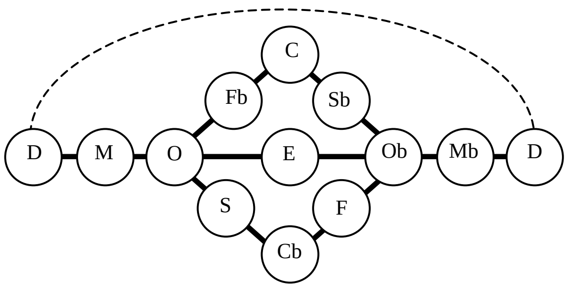

Figure 8. Conceptual neighborhood structure describing temporal relations between occurrences of cycles.

Based on the conceptual neighborhood of temporal relations, we define a second criterion for selecting the best configuration. The second criterion selects among the characteristic sets of correlations that of satisfy the first criterion the set that has greatest (or least) frequency of conceptual neighbors. For example, if it is desirable to maximize the temporal relation *disconnected* between *Bus 1* and *Bus 2*, and the temporal relation *disconnected* does not occur in the set of correlations, then the closest configuration is the one with greatest frequency of the relations *met* and *metBy* (i.e., the conceptual neighbors of the relation *disconnected*). Applying the criterion of conceptual neighborhoods results in the first configuration being selected as the “closest” one that satisfies the intended relation (Figure 7).

The process of breaking a tie with conceptual neighborhoods is recursive in the sense that it can be extended for different degrees of conceptual neighborhoods. Thus, if the immediate conceptual neighborhood is not sufficient to distinguish among different configurations, the second-degree conceptual neighborhood can be used, and so forth. If the highest level of conceptual neighborhood is reached without discerning a unique configuration, we consider that these configurations are indistinguishable with respect to conceptual neighborhood and every configuration satisfies the intended relation.

The temporal configuration between the buses selected by the criteria of frequency of temporal relations is actually the same configuration shown in Figure 3 (i.e., the configuration selected by a person taking into account the city’s policy and the temporal characteristics of the buses behavior). In the latter case, the temporal relationship between the buses was registered in the model through a temporal constraint relating the midpoints of an occurrence of each cyclic interval (i.e., the temporal constraint `centered`). The model with cyclic constraints, however, is more robust (i.e., the model is able to react to a modification in the characteristic set of the cyclic intervals and still keep known relationships between the cyclic events). Consider, for example, the temporal characteristics of *Bus 2's* behavior when some streets on the route of the bus are closed (Figure 4). Considering the temporal characteristics of *Bus 1* and the new temporal characteristics of *Bus 2*, a new period of equivalence can be computed. In this case, the extent of the new period of equivalence is enough to cover one period of *Bus 1* and three periods of *Bus 2*. Thus, the process of incrementing and decrementing the start point of one cyclic interval by one unit of time yields the set of all possible characteristic sets of correlations between *Bus 1* and the new temporal configuration of *Bus 2* (Figure 9).

Since the temporal constraint registered in the model is of the kind `maximizeDisconnected`, the process of selecting the best configuration from a characteristic set of correlations begins with sets of correlations with the greatest number of temporal relations *disconnected*. As in the previous case, the temporal relation *disconnected* does not occur in the set of possible characteristic sets of correlations (Figure 9). Thus, maximizing *disconnected* is satisfied by all possible configurations. Considering the frequency of conceptual neighborhoods (i.e., *met* and *metBy*), results in the first and the second configuration being selected (Figure 9). The process continues with the second degree of conceptual neighborhood (i.e., *overlaps* and *overlappedBy*) to break the tie between these two configurations. Considering the second degree of conceptual neighborhood, the first two configurations are still undistinguishable. This process continues up to the highest degree of conceptual neighborhood without being able to distinguish the first or second configurations. Thus, in this case we consider that both configurations satisfy the intended cyclic constraint and either one can be chosen.

Although the fourth configuration is also a possible configuration of the bus schedules that satisfies the city's policy in the financial district, the criterion of frequency of temporal relations ruled out this configuration. The main reason for that is because the characteristic set of correlations reflects only binary temporal relations (i.e., relations between two occurrences of cyclic intervals). Therefore, it is impossible to distinguish, for example, between the incidence of the relations *overlaps* and *overlappedBy* in the fourth and sixth configurations.

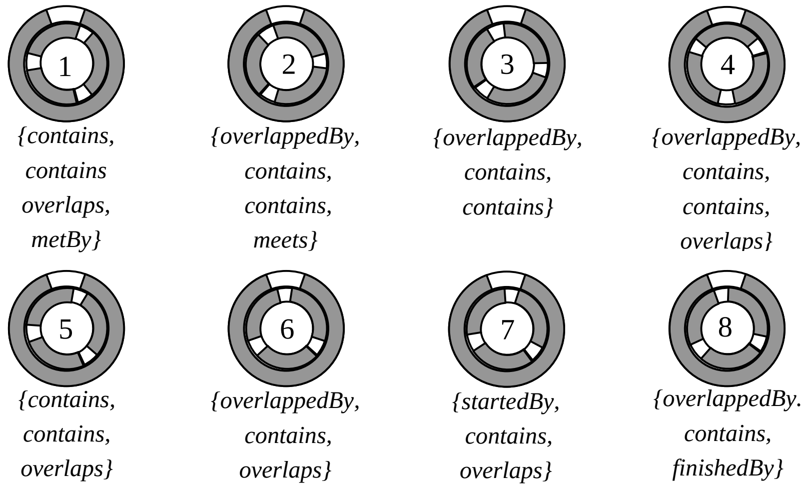

Figure 9. Possible characteristic sets of correlations between *Bus 1* and the new configuration of *Bus 2*.

Among all possible temporal configurations of the buses, the sixth, seventh and eighth configurations are the worst configurations considering the city policy regarding the schedule of the buses. These configurations have the longest periods of simultaneous inactivity of the buses. Coincidently, the approach of relating the buses' cyclic intervals with an instance of a temporal constraint that relates only two occurrences of the cycle, discussed early in this paper, results in the selection of the sixth configuration. It highlights the fact that using such instances of temporal constraints to related cyclic intervals can lead not only to a selection of a undesirable configuration, but to a selection of a configuration that represents the worst-case scenario.

6. CONCLUSIONS AND FUTURE WORK

This paper introduces a model to treat temporal relationships between cyclic events, a special category of events commonly encountered in GIS applications. The proposed model is based on a structure that captures the set of possible temporal relations between occurrences of cyclic events. This structure captures also the frequency in which each temporal relation occurs when all occurrences of the cyclic events are taken into account. Based on such a framework, this paper proposes a new set of temporal constraints to capture relationships between cyclic events (i.e., cyclic constraints) and a rationale to select among different temporal arrangements of the cyclic intervals, the one that best represents the imposed temporal constraint. This paper shows that the model of cyclic constraints is more robust than the usual approach of relating two intervals. This model is able to express the complex relationships that can exist between cyclic events and react to a modification in the temporal characteristic of cyclic events while maintaining a known or desired relationship.

Although the proposed model proves more robust than the usual approach of relating two intervals, this model could be extended by incorporating some metric information about the temporal relations between two cyclic events. This kind of information has the potential to differentiate distinct configurations that are considered the same when only qualitative measures are taken into account. In the cases in which the characteristic set of correlations is formed by temporal relations *disconnected*, *overlaps*, *contains*, and their converses, for example, an increment or decrement in one cyclic interval's occurrences do not necessarily change the characteristic set of correlations. For these relations, the temporal location of the intervals can vary in a range of values and still keep the same temporal relation. Thus, these different configurations of sets of correlation are undistinguishable in respect to the frequency in which each correlation occurs. In order to distinguish a certain configuration among set of correlations with the same frequency of these temporal relations, a qualitative criterion (i.e., a weight) for these temporal relations is needed.

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