

# GeoInfo 2011

## Open source implementation of the Multiplicatively Weighted Voronoi Diagram as a TerraView plugin

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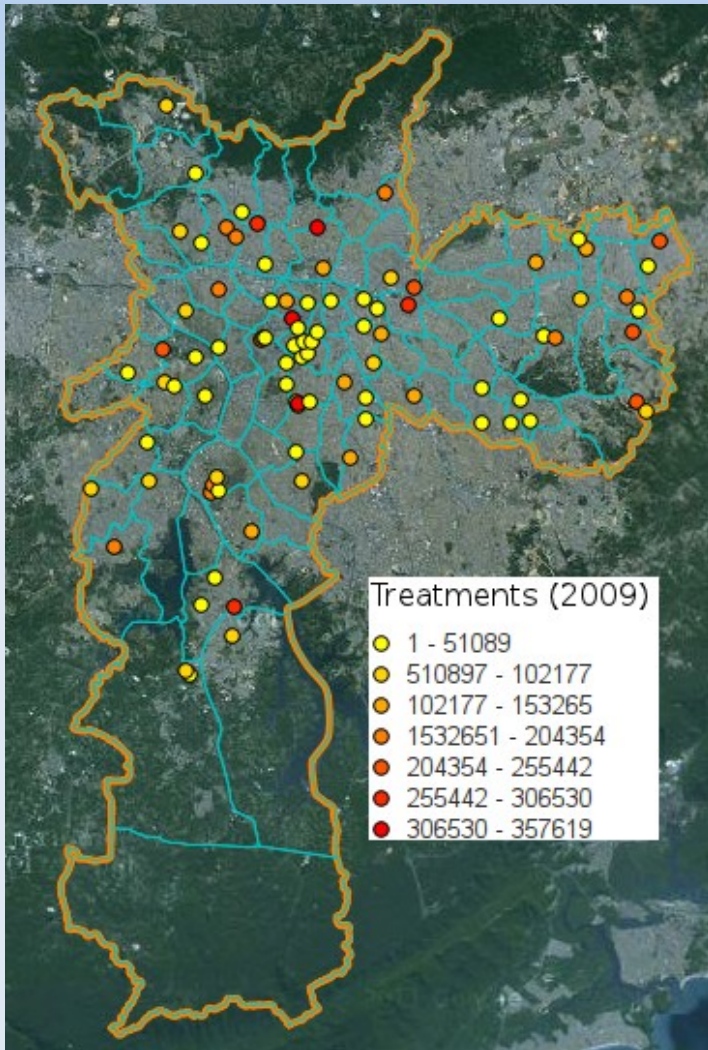
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# Region assignment problem



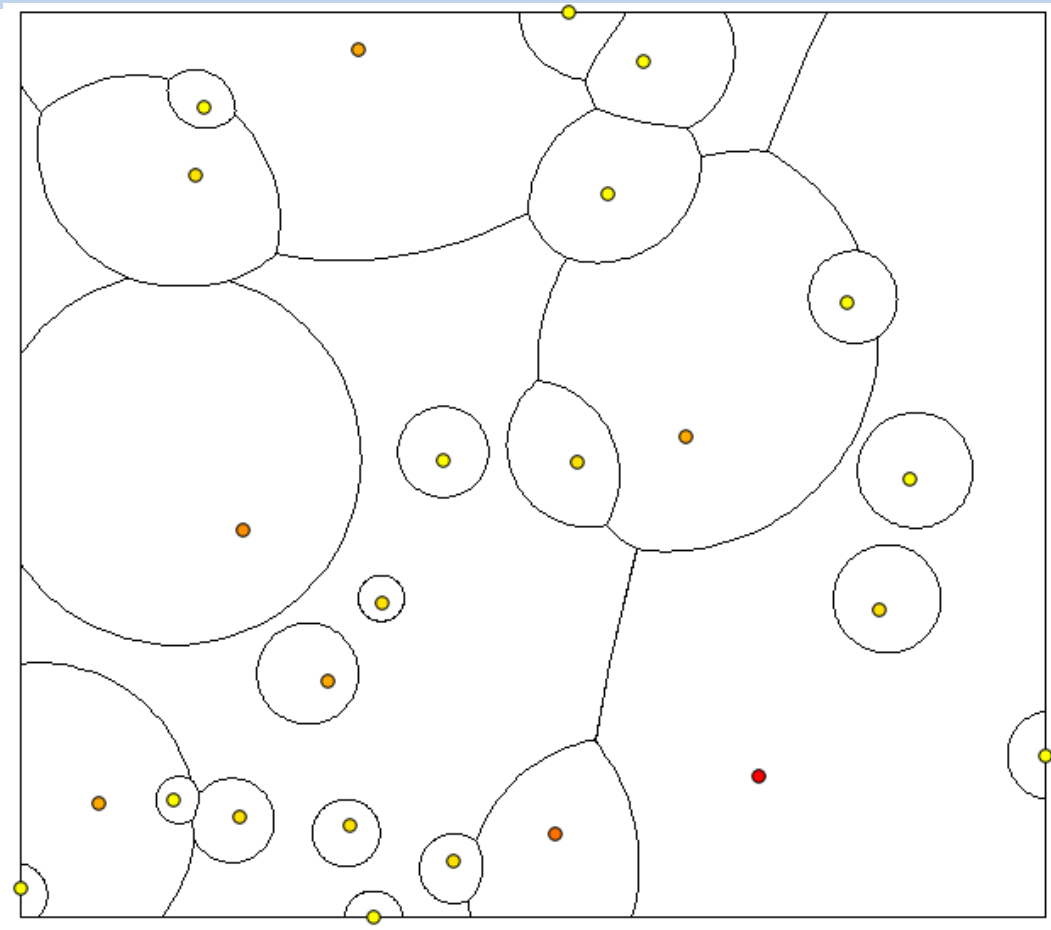
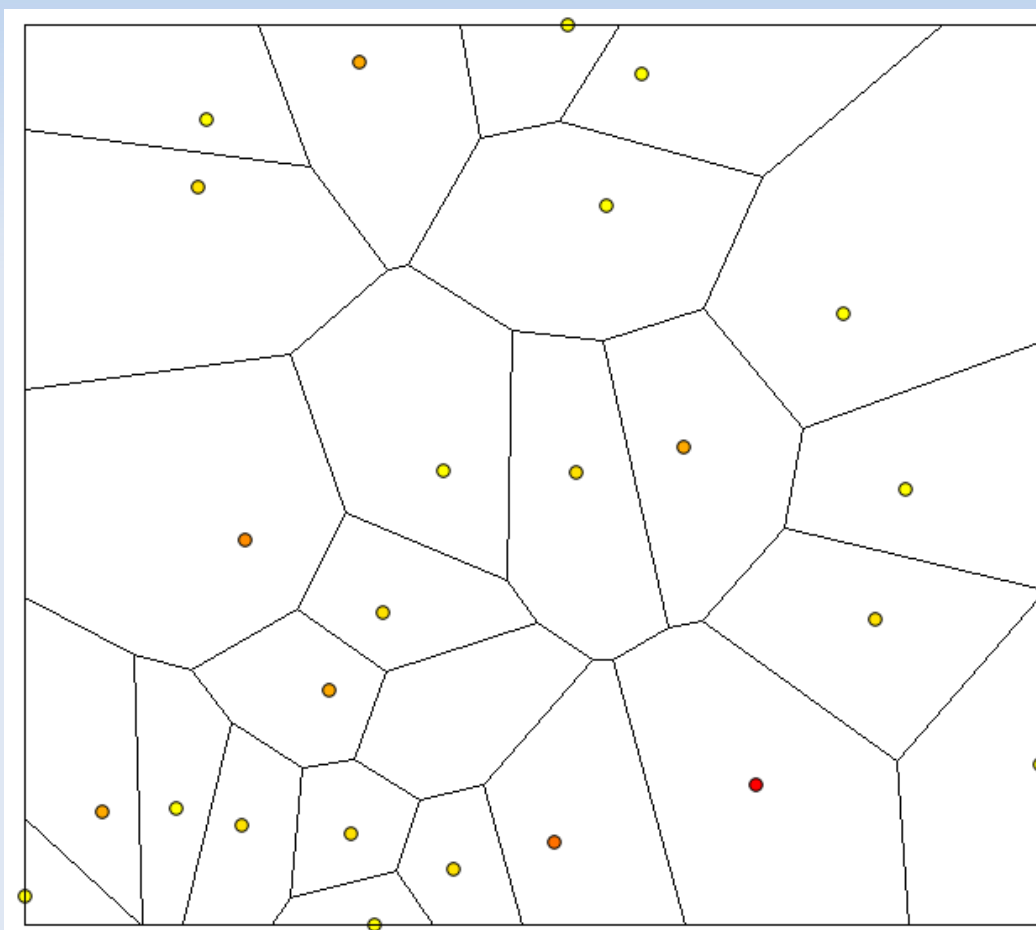
São Paulo's public health centers

- Disaggregation
  - SUS hospitals
  - São Paulo – SP
  - Emergency health care
  - Districts
- Multiplicatively Weighted Voronoi Diagram
  - [Rezende et al, 2000]

# Weighted Voronoi Diagram

Ordinary Voronoi Diagram

Multiplicatively **Weighted** Voronoi Diagram



# Applications

[Boots, 1986]

- Region assignment model
  - Educational centers
  - Logistic districting
  - Health centers
- Growth Model
  - Antenna signals
  - Quickest neighbor diagram

## Weighted distance

[Aurenhammer and Edelsbrunner, 1984]

$$d_w(x, p_i) = \frac{d_e(x, p_i)}{w_i}$$



$$time = \frac{distance}{speed}$$

# Appoloniu's Circle

Weighted distance:

$$d_w(x, p_i) = \frac{d_e(x, p_i)}{w_i}$$

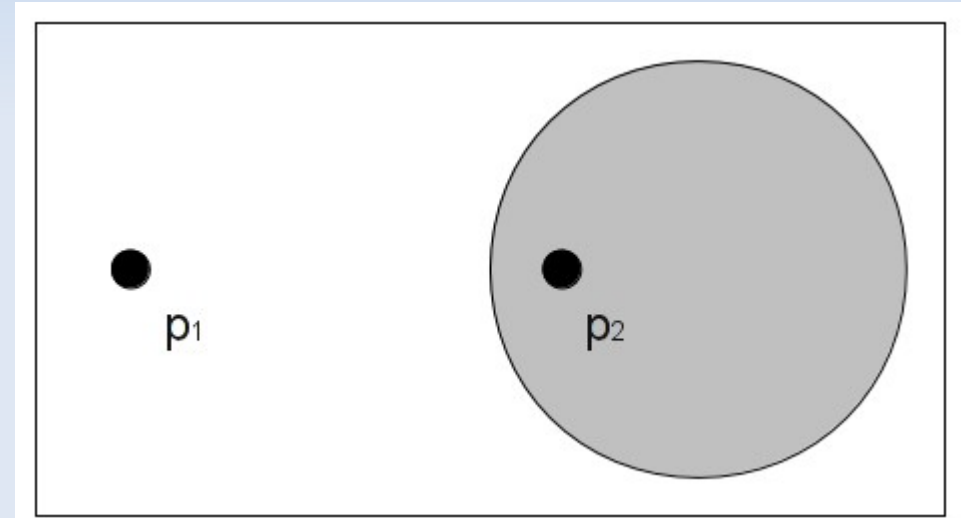


Boundaries:

$$\frac{d_e(x, p_1)}{d_e(x, p_2)} = \frac{w_1}{w_2}$$

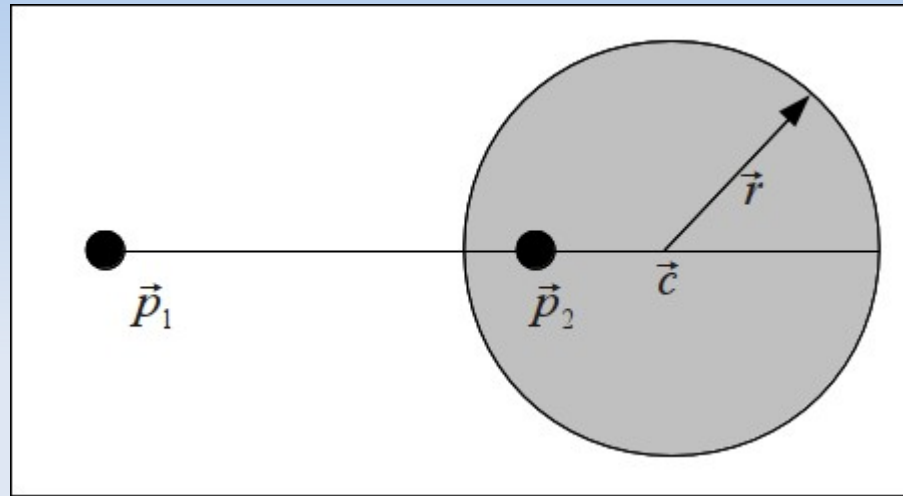


$w_1 > w_2 \rightarrow p_1$  dominates  $p_2$



[Aurenhammer and Edelsbrunner, 1984]

# Appoloniu's Circle



Center

$$\vec{c}_1 = \frac{w_2^2 \cdot \vec{p}_1 - w_1^2 \cdot \vec{p}_2}{w_1^2 - w_2^2}$$

Radius

$$|\vec{r}| = \frac{w_1 \cdot w_2 \cdot |\vec{p}_1 - \vec{p}_2|}{w_1^2 - w_2^2}$$

[Aurenhammer and Edelsbrunner, 1984]

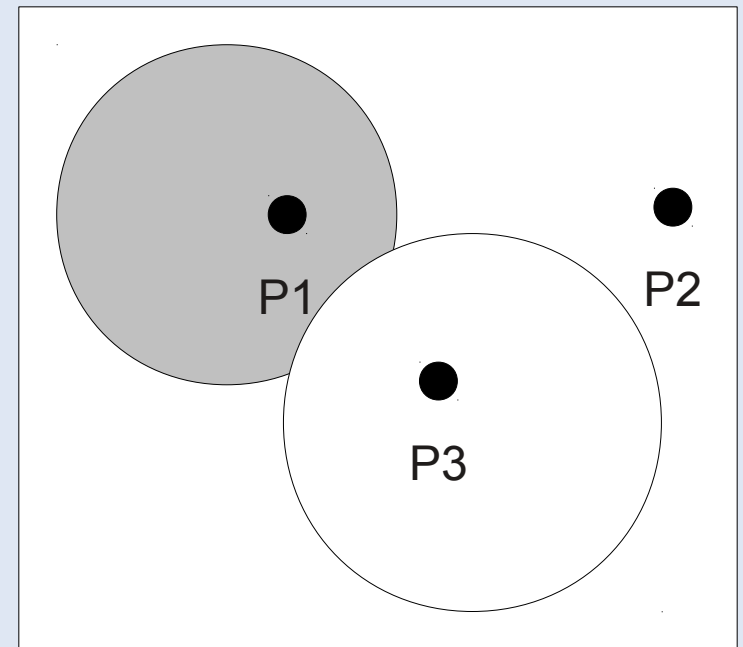
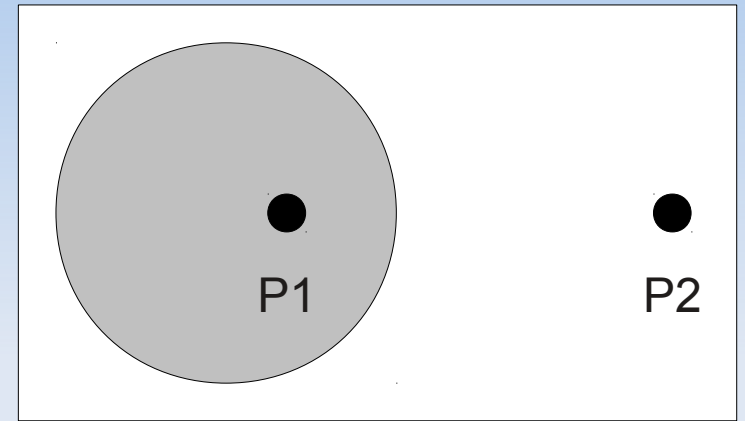
# Proposed Algorithm

For each point

For each other point

Compute the dominance  
circle

Intersect with the current  
dominance area



# Complexity

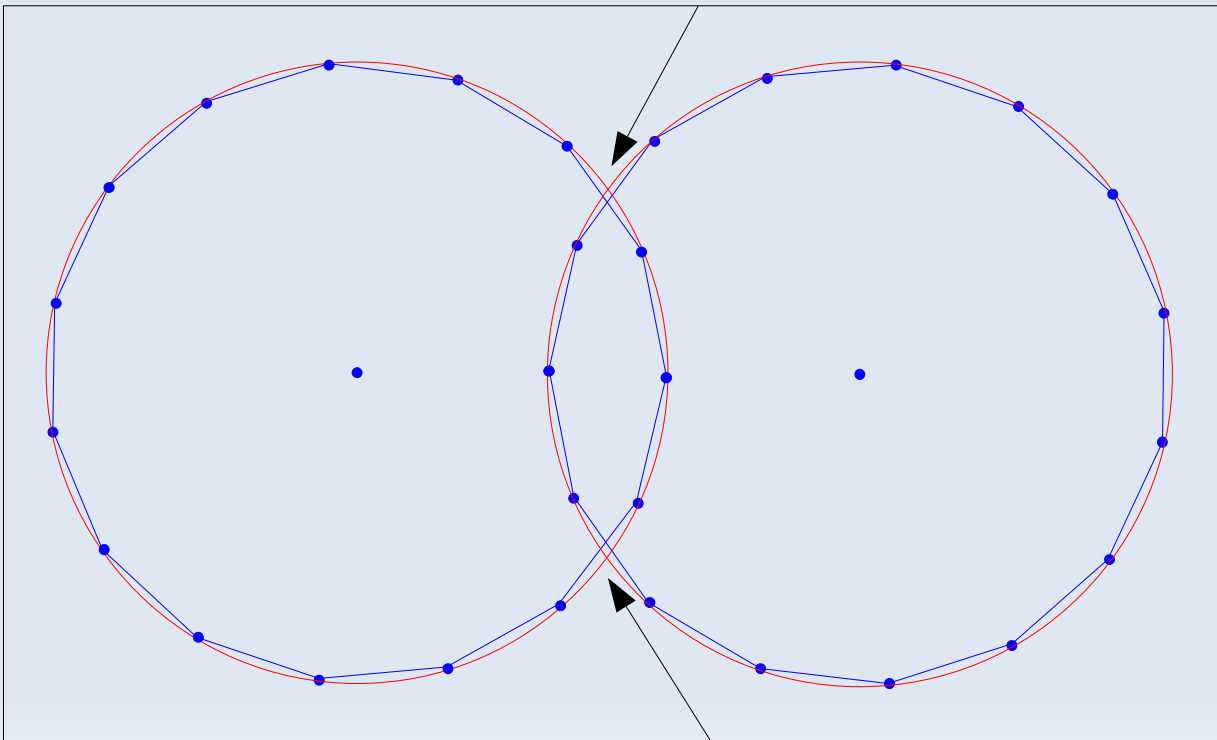
- Computational  $O(n^3)$ 
  - $O(n^2)$  area/circle intersections
  - Each dominance area has at most  $n$  sides.

[Aurenhammer and Edelsbrunner, 1984]

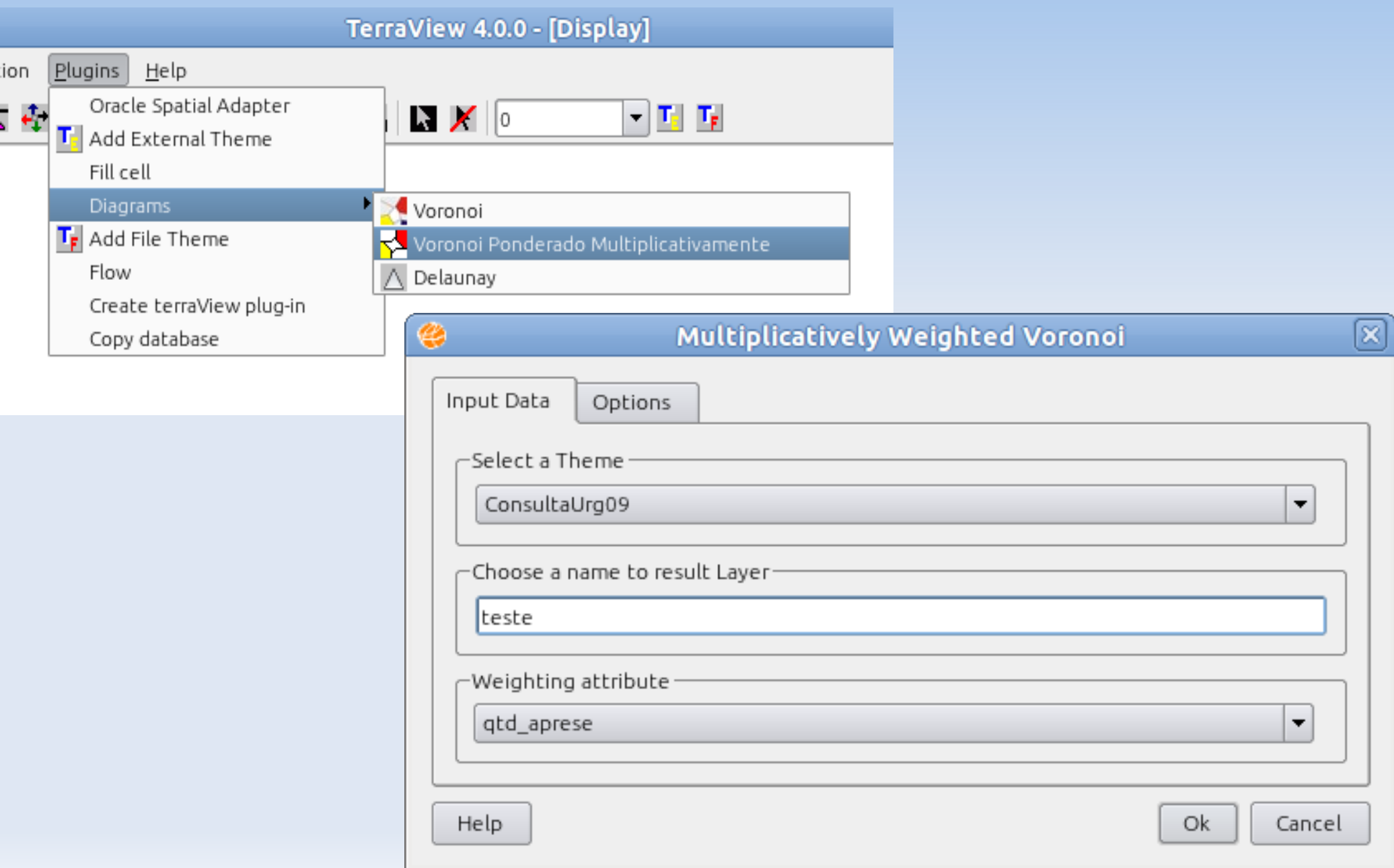
- Memory
  - 1 area
  - 1 circle

# Intersection of Polygons/Circles

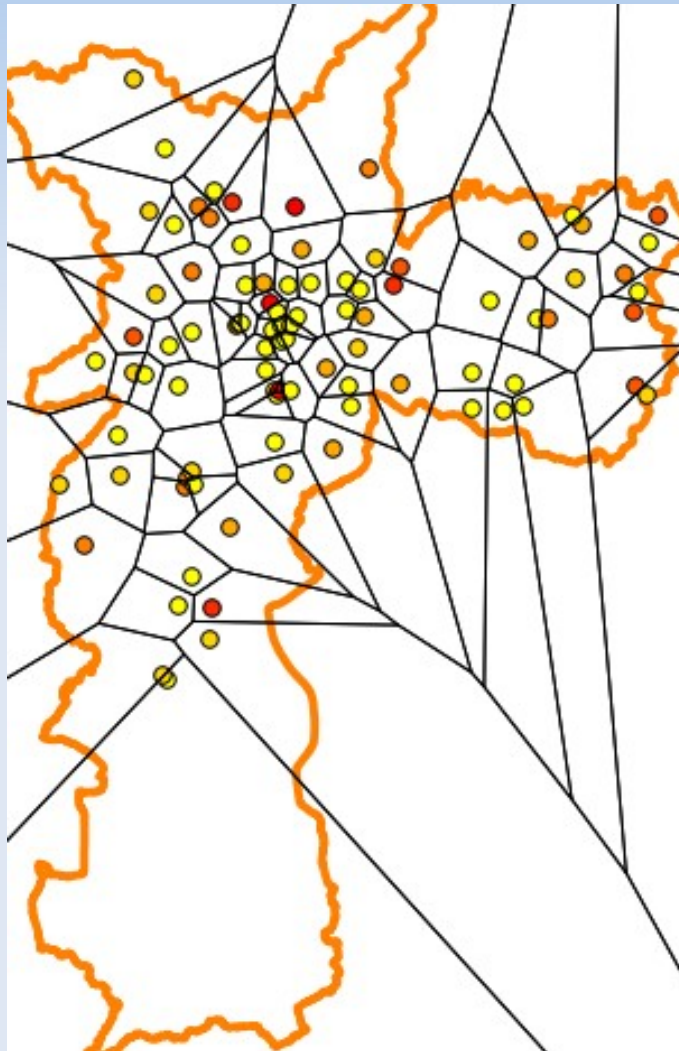
- Polygonize → Intersect
- Intersect → Polygonize
- SQL\MM
  - Polygons (360 sides)
  - CurvePolygon



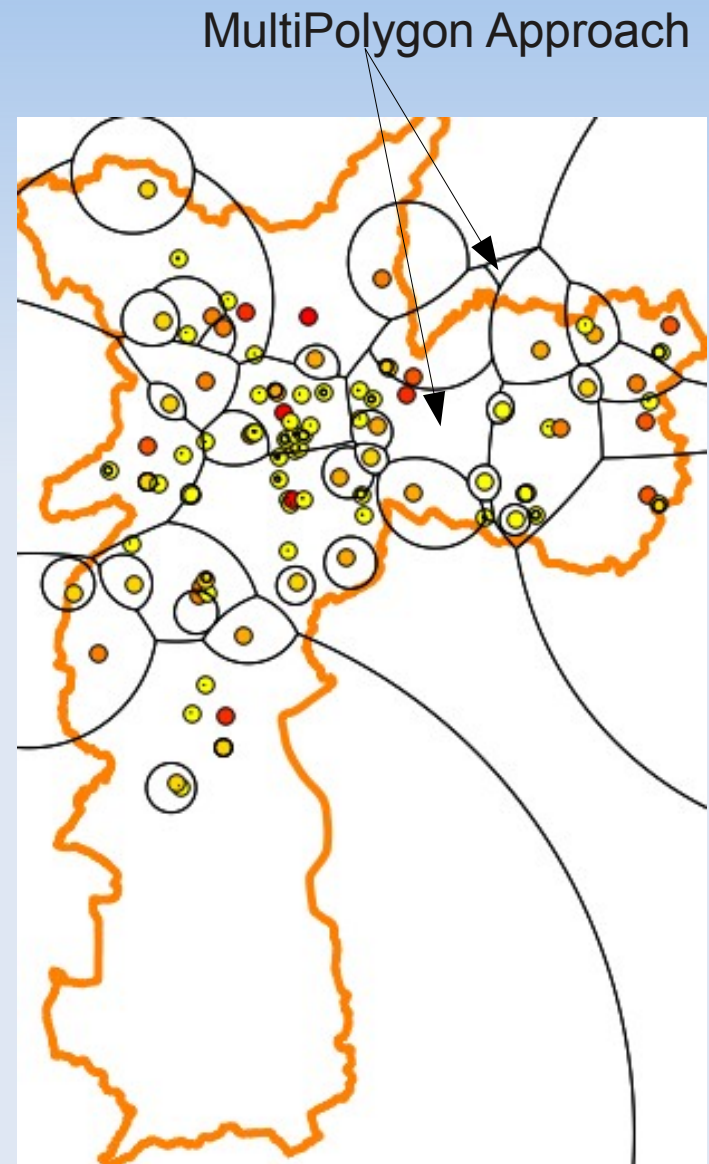
# Interface



# Results



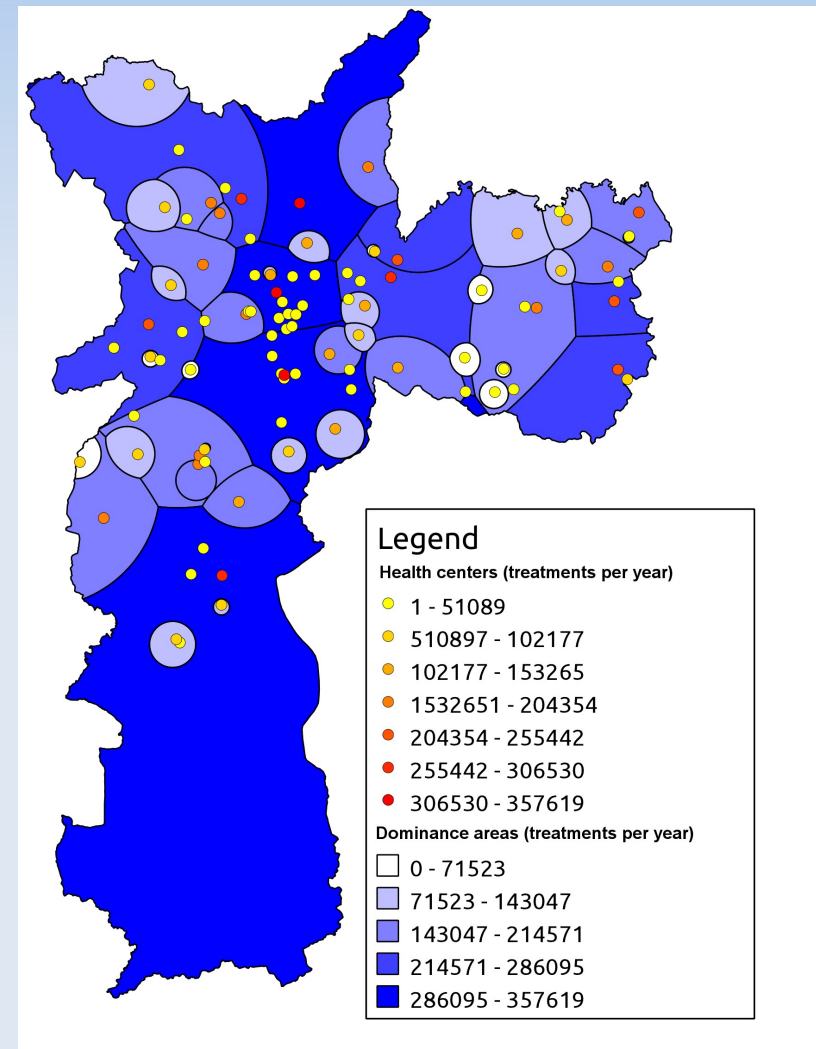
Ordinary Voronoi



Multiplicatively Weighted Voronoi  
Weight: Emergency Treatments in 2009

# Conclusion

- Vector-based algorithm
- Multi-part dominance
- Speed loss
- Intersection accuracy
- MultiPlatform



São Paulos' Health Centers' dominance

# Acknowledgements

- National Institute for Space Research
- Brazilian Army – Geographic Service

# References

- Aurenhammer, F. and Edelsbrunner, H. (1984). **An optimal algorithm for constructing the weighted Voronoi diagram in the plane.** Pattern Recognition, 17(2):251–257.
- Rezende, F., Almeida, R., and Nobre, F. (2000). **Diagramas de Voronoi para a definicao de áreas de abrangência de hospitais públicos no Município do Rio de Janeiro.** Cadernos de Saúde Pública, 16(2):467–475.
- Boots, B. (1986). **Voronoi (Thiessen) Polygons**, volume 45. Geo Books.
- Dong, P. (2008). **Generating and updating multiplicatively weighted Voronoi diagrams for point, line and polygon features in GIS.** Computers & Geosciences, 34(4):411–421.